CDO models: Opening the black box

Large homogeneous pool model

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Structured credit research

Dresdner Kleinwort

Putting the theory to work

Infinite, homogeneous portfolio

- In the first part of our series we release the Large Homogenous Pool Model in the standard version as well as a version using the Gauss-Hermite Integration technique
- This publication has been structured as a user guide to be used in conjunction with the excel-based model. Whilst we do briefly touch upon the main theoretical concepts, we do not go into detailed explanations and proofs, as this information has been widely discussed and is readily available. Instead, we focus on how to implement the theory and apply the models
- Given the simplified assumptions behind this model, it is not a pricing tool for CDO tranches but instead is the first step to allow the user to appreciate the impact of key parameters such as correlation, recovery and spread on the value of a specific tranche
- Additionally, as the pool is considered to have an infinite and identical number of obligors, aspects such as idiosyncratic risk are not specifically treated. We will relax and analyse these points in the upcoming models

Large Homogeneous Pool Model:

https://research.dresdnerkleinwort.com/document/FILE.pdf?REF=241839

Large Homogeneous Pool Model with Gauss-Hermite Integration:

https://research.dresdnerkleinwort.com/document/FILE.pdf?REF=241841

Model components at a glance…

Deal Parameters

Discount Rate (%)	5.00%
Coupon payment frequency (p.a.)	4
Average Recovery (%)	40.00%
Index Spread (bps)	100.00
Hazard Rate ~ Clean spread	166.67
Cumulative Default Probability	8.17%
Total Portfolio Notional	1,000,000
Value Date	04-Sep-08
Maturity Date	20-Sep-13
Next Coupon Date	20-Sep-08
Horizon (as a year fraction)	5.1167
Maturity in months	60
Correlation	20%

Key Model Outputs

	Upfront	Running Spread (bps)	DV01
0% - 3%	58.25%	500.0	2.27
3% - 6%		957.3	3.72
6% - 9%		428.8	4.17
9% - 12%		208.4	4.34
12% - 22%		55.6	4.45
22% - 100%		0.7	4.39
Index		**100.0**	**4.30**

Numerical Integration

Factor	-0.1	0.0	0.1
Integral	0.0396953	0.0398942	0.0396953

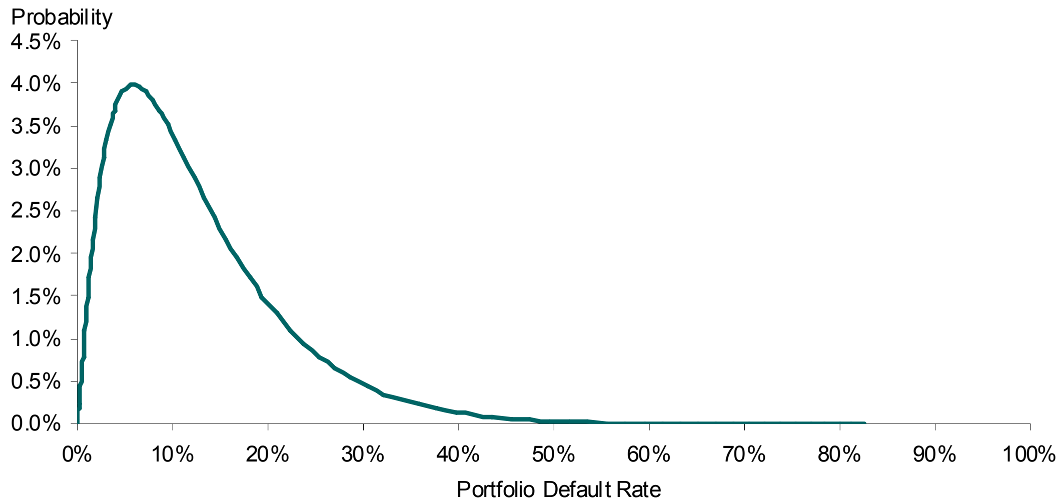
Portfolio default distribution, conditional on factor

1	0.02%	0.02%	0.02%
2	0.23%	0.20%	0.17%
…	…	…	…
20	6.209%	5.620%	5.077%
21	6.579%	5.963%	5.393%

Tranche loss, conditional on factor

1	0.46%	0.38%	0.31%
2	4.59%	3.92%	3.34%
…	…	…	…
20	100.00%	100.00%	100.00%
21	100.00%	100.00%	100.00%

Portfolio default rate distribution



The main model inputs

Spreadsheet screenshot

Deal Parameters

Discount Rate (%)	5.00%
Coupon payment frequency (p.a.)	4
Average Recovery (%)	1 40.00%
Index Spread (bps)	100.00
Hazard Rate ~ Clean spread	166.67
Cumulative Default Probability	8.17%
Total Portfolio Notional	1,000,000
Value Date	04-Sep-08
Maturity Date	20-Sep-13
First Coupon Date	20-Sep-08
Horizon (as a year fraction)	5.1167
Maturity in months	60
Correlation	2 20%

Based on the spread and average recovery (R) we can estimate the hazard rate(λ) from the clean spread:

$$\lambda = \frac{spread}{(1-R)}$$

The cumulative default probability to any time t (PD_t) can then be calculated as

$$PD_t = 1 - SP_t = 1 - e^{-\lambda t}$$

Payment dates:
20th of March, June, Sep, Dec

Although not needed when pricing single name CDS or CDS indices, the correlation parameter is the key assumption for pricing CDO tranches.

Correlation drives the shape of the loss distribution, and hence the risk allocation between the tranches.

Underlying assumptions

- **Key portfolio assumption**: In the large homogeneous pool framework, the CDO collateral pool is assumed to be infinite in size and homogeneous. Homogeneity implies that the index is equivalent to a single name CDS with identical spread and recovery
- In addition, without loss of generality, we assume flat CDS spreads and a flat interest rate curve for discounting
- In line with market practices, we use quarterly CDS coupon payments. The model provides the flexibility to price contracts with maturities up to 60 months, which is generally equivalent to 21 coupons payments. The first payment is with respect of the part quarter to the first fixed coupon date

The defining relationship between spreads and default probabilities

Fee leg, contingent leg and risky annuity

- The fair spread of a CDS contract is calculated by equating the present value of the expected future coupons payments (fee leg) with the present value of the expected loss payable following an event of default (contingent leg)

- The key formula:

$$\underbrace{\underbrace{S\sum_{t=1}^{T} e^{-it}\Delta_t SP_t}_{\text{Coupon leg}} + \underbrace{S\sum_{t=1}^{T} e^{-it}\frac{\Delta_t}{2}(SP_t - SP_{t-1})}_{\text{Accrued coupon on default}}}_{\text{Fee Leg}} = \underbrace{(1-R)\sum_{t=1}^{T} e^{-it}(SP_t - SP_{t-1})}_{\text{Contingent Leg}}$$

Where:

$S = \text{CDS spread}$

$R = \text{Recovery rate}$

Coupon payments, at times $t = 1$ *to* T

$i = \text{risk-free discount rate (p.a.)}$

$\Delta_t = \text{time period between coupon periods}$

$SP_t = \text{Survival probability up to period } t = e^{-\lambda t}$

$SP_t - SP_{t-1} = \text{Marginal default probability}$

- The risky annuity (PV01) can also be calculated using this relationship and can then be used as an estimate of the contract's risky duration (DV01)

$$\sum_{t=1}^{T} e^{-it}\Delta_t SP_t + \sum_{t=1}^{T} e^{-it}\frac{\Delta_t}{2}(SP_t - SP_{t-1}) = \mathrm{PV01} \approx DV01$$

- The CDS Index spread is calculated in a similar manner as for a single CDS. When assuming a homogeneous pool, the index spread and recovery is the same as that of the underlying. For a heterogeneous pool, the index spread is calculated as the DV01-weighted average spread of the portfolio, and the index recovery as the average recovery of the pool

The defining relationship between spreads and default probabilities

Spreadsheet screenshot

$S_t = e^{-\lambda t}$

Δ_t | e^{-it}

Pay date information | **Index calculation using the index spread**

Time Period (t)	Pay Date	Day Count	Discount Factor	Cum. Defaults	Survivals	Marg. Defaults
1	20-Sep-08	0.044	99.78%	0.07%	99.93%	0.07%
2	20-Dec-08	0.253	98.52%	0.49%	99.51%	0.42%
3	20-Mar-09	0.250	97.30%	0.91%	99.09%	0.41%
4	20-Jun-09	0.256	96.07%	1.33%	98.67%	0.42%
5	20-Sep-09	0.256	94.85%	1.75%	98.25%	0.42%
..	…..	…..	…..	…..	….	…..
18	20-Dec-12	0.253	80.43%	7.00%	93.00%	0.39%
19	20-Mar-13	0.250	79.43%	7.39%	92.61%	0.39%
20	20-Jun-13	0.256	78.42%	7.78%	92.22%	0.39%
21	20-Sep-13	0.256	77.43%	8.17%	91.83%	0.39%

Coupon Leg	
4.30	
Accrual on default	
0.009	
Fee leg ≈DV01	**Contingent Leg**
4.31	4.3081%
Running Spread	
1.0000%	

1 Using the previously defined relationship between spreads and survival (default) probabilities, the cumulative default probabilities can be iteratively calculated from market observed spreads

In our model, without loss of generality, we assume flat CDS spreads, and hence given spread S and recovery R, the cumulative default probability, to any time t, is simply:

$$PD_t = 1 - e^{-\frac{S}{1-R}t} = 1 - e^{-\lambda t}$$

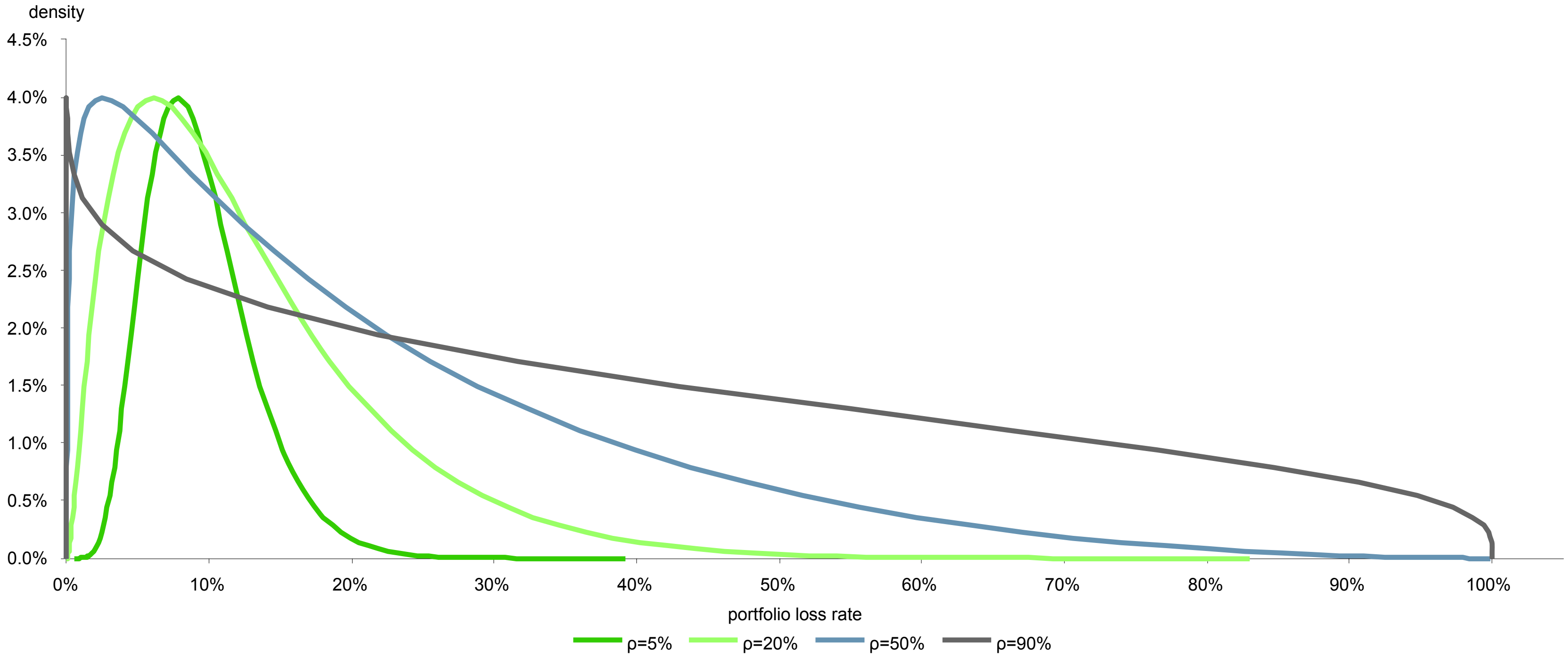
2 Alternatively, the formula can also be applied to a set of cumulative default probabilities, and the running spread can then be calculated as

$$S = \frac{Contigent\ Leg}{Fee\ Leg}$$

When pricing CDO tranches, this is the eventual aim.
For each coupon pay date, once the distribution has been obtained, the survival probabilities for each tranche can be calculated and the same relationship can be applied to obtain the tranche spread.

Moving to a CDS portfolio: correlation is key

Portfolio loss distributions for different levels of correlation



Source: Dresdner Kleinwort Research

- The portfolio's expected portfolio loss (EL) is not influenced by the level of correlation
- However, the correlation between the underlying obligors drives the shape of the portfolio's loss distribution and hence the risk allocation between the tranches
- Therefore, for single tranches correlation is an important parameter when determining the tranche EL and spread. When correlation is high there is a higher probability of zero losses, however the probability of large losses impacting the senior tranches also increases

The one factor model approach

Asset values are correlated via a common factor

- Correlation between the portfolio constituents means that the default probability of one asset is dependant on the other assets in the portfolio
- Such dependent probabilities increase the mathematical complexity and therefore, rather than directly specifying correlation between the assets, correlation is more efficiently introduced via a factor model where the correlation between each firm and a common macroeconomic factor is modelled
- Each asset (firm) is then correlated with this common market factor. Therefore, given a particular realisation of the market factor, the individual asset default probabilities are then independent, and hence more easily handled
- A one factor model is commonly used, where the value of the an asset is modelled assuming a linear relationship with the market factor

$$V_n(t) = \sqrt{\rho_n} M(t) + \sqrt{1 - \rho_n} \varepsilon_n(t)$$

  - The common factor M determines the systematic risk and can be interpreted as the state of the economy
  - The idiosyncratic factor ε_n can be understood as a firm specific risk component
  - Both M and all ε_n are assumed to follow independent standard normal distributions (Φ~(0,1)) and therefore the firm value V_n is also a standard normal
- The asset correlation parameter ρ determines the extent to which the firm value depends on the common and the idiosyncratic risk factors. For simplicity, we assume the correlation with the common market factor is identical for all assets:

$$\rho_n = \rho \quad \forall\, n$$

The one factor model approach

Independent conditional default probabilities

- Using the simplest Merton framework, a default occurs if a firm's value falls below a certain default barrier

$$PD_n(t) = P\left[V_i(t) \leq K_i(t)\right]$$

- The joint default behaviour of two firms is hence modelled based on asset values, and the input parameter is therefore the asset value correlation and NOT the default correlation

- Based on market observed CDS spreads the default probability to time t can be calculated (as before), and the default barrier for each asset can then be backed out. As we are assuming a homogeneous pool, for all assets the CDS spread and hence the default probabilities and default barrier are identical and calculated as:

$$K(t) = \Phi^{-1}\left(PD(t)\right)$$

- Finally, the cumulative probability of default conditional on a given realisation of the common factor *M* is given by

$$PD\left(t \middle| m\right) = \Phi\left(\frac{K(t) - \sqrt{\rho}\, m}{\sqrt{1-\rho}}\right) \Longrightarrow PD(t) = \sum_{\forall m} PD\left(t \middle| m\right) P\left(M = m\right)$$

- As discussed earlier, for a given realisation of the common factor *m* these probabilities are independent, and as we assume a homogeneous pool, they are also identical for all assets

- At portfolio level, the assumption in the case of a large homogenous pool (i.e. an infinite number of assets) at the same time gives the proportion of the portfolio defaulting by time *t* conditional on *m*

The one factor model in Excel

Spreadsheet screenshot

$$PD(t) = \sum_{\forall m} PD(t|m)P(M = m)$$

The PD_t is calculated using the conditional default probabilities and compared with the initial PD_t input as a final check

Realisation of common factor M. Integration from -5 to +5 in steps of 0.1

P(M=m)=*f(m)**step size

For m= -4.5
P(M=-4.5)= ϕ(-4.5)*0.1

Notation used for standard normal:

ϕ(x)=f(x)
Φ(x)=F(x)=P(X≤x)

Numeric[illegible]ion

1 2

Common factor (m)	-5.0	-4.9	-4.8	-4.7	-4.6	-4.5	-4.4	-4.3
Integral	1.48672E-07	2.43896E-07	3.9613E-07	6.36983E-07	1.01409E-06	1.59837E-06	2.49425E-06	3.85352E-06

5 3 4

norminv(pd) = default barrier	[pd(t)]	Check	Time Period	Portfolio default distribution, conditional on factor (m)							
-3.178	[illegible].074%	0.00%	1	14.604%	13.489%	12.434%	11.438%	10.500%	9.619%	8.794%	8.023%
-2.580	0.494%	0.00%	2	35.034%	33.199%	31.405%	29.653%	27.947%	26.290%	24.686%	23.135%
-2.362	0.908%	0.00%	3	44.385%	42.417%	40.469%	38.544%	36.647%	34.782%	32.952%	31.163%
-2.218	1.329%	0.00%	4	50.823%	48.828%	46.837%	44.853%	42.882%	40.929%	38.998%	37.094%
-2[illegible]109	1.748%	0.00%	5	55.661%	53.680%	51.690%	49.696%	47.702%	45.71[illegible]	43.738%	41.776%
-[illegible]21	2.161%	0.00%	6	59.480%	57.531%	55.564%	53.582%	51.592%	49.598[illegible]	47.604%	45.617%
-[illegible]8	2.568%	0.00%	7	62.613%	60.704%	58.769%	56.813%	54.840%	52.854%	[illegible]862%	48.868%

$$K(t) = \Phi^{-1}(PD(t))$$

The default barrier for time period 3 is calculated using the default probability of the asset

$$K(3) = \Phi^{-1}(0.91\%)$$

$$PD(t|m) = \Phi\left(\frac{K(t) - \sqrt{\rho}\,m}{\sqrt{1-\rho}}\right)$$

The cumulative probability of default conditional on a common factor M of -4.5

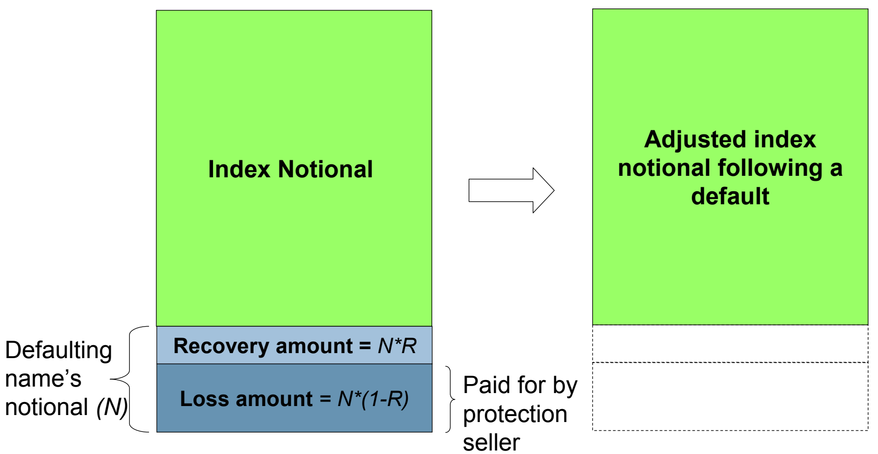
$$PD(t = 3|m = -4.5) = \Phi\left(\frac{-2.362 - \sqrt{\rho}(-4.5)}{\sqrt{1-\rho}}\right)$$

Default treatment for indices vs. tranches (don't forget the recovery)

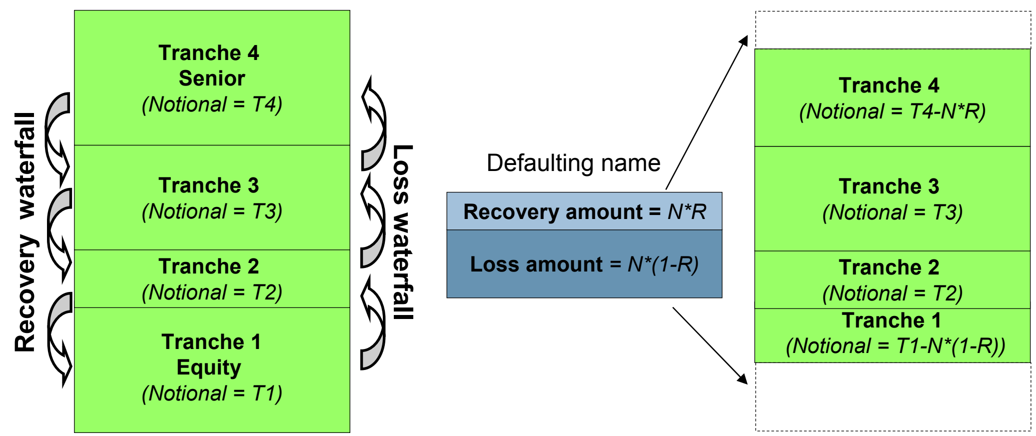
Losses to equity, recovery to super senior

- For a single name CDS, a default event results in the payment of the loss amount from the protection seller to the protection buyer and the contract is then cancelled
- At an index level, each obligor has a fixed notional exposure. If one of the portfolio names defaults, the resulting loss is similarly paid for by the protection buyer
- However, the index contract is not cancelled. Instead the index notional is reduced by the defaulting name's notional and future coupons are then based on this adjusted value
- When dealing with tranches, to ensure no arbitrage, the sum of the tranche notionals should always be equal to the index notional. Therefore, following a default the sum of the tranche notionals should also decrease by the defaulting name's notional
- However, as the equity tranche is the first loss piece, the recovered amount is not reduced from the equity. Instead, losses are passed through from the equity tranche up the capital structure and recovery from the most senior tranche downwards
- From our experience, we find that the recovery waterfall generally tends to be ignored BUT it is an important element of the model and should always be included

Default treatment for a CDS index



Default treatment for an CDS tranches



Allocating portfolio losses and recoveries to tranches

Starting with the conditional default probability, portfolio losses are allocated to tranches

- For the infinite case, the conditional default probabilities (for each time period *t* and market factor *m*) also tell us the proportion of the portfolio that has defaulted
- This default adjusted by the recovery gives us the portfolio loss, which is then allocated sequentially starting from the equity tranche. The resulting recovery is allocated from the super senior tranche
- If portfolio losses exceed a tranche's attachment point, then the tranche will suffer a loss. The tranche is fully wiped out if the portfolio loss exceeds its detachment point. In addition, because of the leverage inherent in tranches, tranche losses are a multiple of its width
- Therefore for an equity tranche with width 3%, a 1% portfolio loss will result in 33.33% (=1% / 3%) loss for the equity tranche
- Bearing this relationship in mind, given a portfolio loss of L_{Index}, the calculation of the loss associated with any tranche is straightforward:

$$L_{Tranche} = \frac{\min\left[\max(0, L_{Index} - Attachment), Tranche\ Width\right]}{Tranche\ Width}$$

- Similarly, portfolio recovery is allocated based on the tranche width and its detachment point. Only when recoveries exceed a tranche's detachment point, the tranche notional will be reduced by any excess recovery:

$$R_{Tranche} = \frac{\min\left[\max(0, R_{Index} - (1 - Detachment)), Tranche\ Width\right]}{Tranche\ Width}$$

Allocating portfolio losses and recoveries to tranches

Spreadsheet screenshot

Common factor (m)	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3
Integral	0.0381388	0.039104	0.039695	0.039894	0.039695	0.039104	0.038139

Portfolio default distribution, conditional on factor (m)

Time Period							
1	0.033%	0.028%	0.023%	0.019%	0.016%	0.013%	0.011%
2	0.312%	0.268%	0.230%	0.196%	0.167%	0.142% **1**	.121%
3	0.637%	0.552%	0.478%	0.413%	0.356%	0.306%	0.263%
…	…	…	…	…	…	…	…
19	7.101%	6.447%	5.841%	5.280%	4.763%	4.287%	3.851%
20	7.529%	6.845%	6.209%	5.620%	5.077%	4.576%	4.115%
21	7.958%	7.243%	6.579%	5.963%	5.393%	4.867%	4.383%

Tranche loss, conditional on factor (m)

	Equity	0.67%	0.55%	0.46%	0.38%	0.31%	0.26%	0.21%
0%	3%	6.25%	5.36%	4.59%	3.92%	3.34%	2.84% **2**	2.41%
		12.73%	11.05%	9.56%	8.26%	7.12%	6.12%	5.25%
		…	…	…	…	…	…	…
		100.00%	100.00%	100.00%	100.00%	95.26%	85.74%	77.01%
		100.00%	100.00%	100.00%	100.00%	100.00%	91.52%	82.31%
		100.00%	100.00%	100.00%	100.00%	100.00%	97.33%	87.65%

Tranche recovery, conditional on factor (m)

	Super Senior	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
22%	100%	0.16%	0.14%	0.12%	0.10%	0.09%	0.07% **3**	0.06%
		0.33%	0.28%	0.25%	0.21%	0.18%	0.16%	0.13%
		…	…	…	…	…	…	…
		3.64%	3.31%	3.00%	2.71%	2.44%	2.20%	1.97%
		3.86%	3.51%	3.18%	2.88%	2.60%	2.35%	2.11%
		4.08%	3.71%	3.37%	3.06%	2.77%	2.50%	2.25%

As shown before, the conditional default probabilities are calculated as:
Shown, here for t=3, m=0.3

$$PD\left(t=3 \middle| m=0.3\right)=\Phi\left(\frac{-2.362-\sqrt{\rho}\,0.3}{\sqrt{1-\rho}}\right)$$

Given a recovery of 40%, a portfolio default of 0.263% translates into a portfolio loss of 0.158% and portfolio recovery of 0.105%

Portfolio loss of 0.158% implies an equity tranche loss of

$$L_{Tranche}=\frac{\min\left[\max(0, 0.158\%-0\%), 3\%\right]}{3\%}=\frac{0.158\%}{3\%}$$

The equity tranche is fully wiped out only once portfolio losses exceed 3%, and the next tranche then gets affected

Portfolio recovery of 0.105% implies a super senior tranche notional reduction of

$$R_{Tranche}=\frac{\min\left[\max(0, 0.105\%-(1-100\%)), 78\%\right]}{78\%}$$

$$=\frac{0.105\%}{78\%}$$

Tranche pricing

From conditional to unconditional tranche loss and recovery

- For each CDO tranche we now have two matrices showing, for each time period *t* and market factor *m*, the conditional cumulative tranche loss and tranche recovery
- To find the unconditional cumulative loss and recovery for each time period, we simply multiply the conditional value by the probability that the market factor equals *m.* (Similar to step 5 on page 9)

Spreadsheet screenshot

Unconditional cumulative loss by time *t*

$$TL_{Eq}(t) = \sum_{\forall m} L_{TrancheEq}(t|m)P(M = m)$$

Calculated by multiplying the row of conditional tranche loss by time *t=1* with *P(M=m)*

Unconditional cumulative recovery by time *t*

$$TR_{SS}(t) = \sum_{\forall m} R_{TrancheSS}(t|m)P(M = m)$$

Calculated by multiplying the row of conditional tranche recovery by time *t=1* with *P(M=m)*

Model outputs for specific tranches

	Equity					…	Super Senior				
Time period	Notional (End Period)	Marg. Loss	Marg. Recoveries	Cum. Recoveries	Cum. Loss (1)	…	Notional (End Period)	Marg. Loss	Marg. Recoveries	Cum. Recoveries (2)	Cum. Loss
1	98.52%	1.48%	0.00%	0.00%	1.48%	…	99.96%	0.00%	0.04%	0.04%	0.00%
2	90.40%	8.12%	0.00%	0.00%	9.60%	…	99.75%	0.00%	0.22%	0.25%	0.00%
…	…	…	…	…	…	…	…	…	…	…	…
20	22.35%	1.67%	0.00%	0.00%	77.65%	…	95.98%	0.01%	0.20%	3.99%	0.03%
21	20.84%	1.50% (3)	0.00%	0.00%	79.16%	…	95.77% (4)	0.01%	0.20%	4.19%	0.04%

Marginal loss for time period *t*

$$MarLoss_{Eq}(t) = TL_{Eq}(t) = TL_{Eq}(t-1) = 79.16\% - 77.65\%$$

Similar calculation for marginal recovery

End period tranche notional is simply the difference between the notional at the start of the period and the marginal loss and marginal recovery for that period

Tranche pricing: equivalent to pricing single name CDS

Spreadsheet screenshot

Similar to a single name CDS, the coupon leg of a tranche is calculated based on the tranche survival likelihood, indicated by the end period notional

$$CouLeg = \sum_{t=1}^{T} e^{-it} \Delta_t Notional_t$$

The accrued coupon on default is a function of the tranche notional lost in any period

$$AccDef = \sum_{t=1}^{T} e^{-it} \frac{\Delta_t}{2} (Notional_t - Notional_{t-1})$$

$$= \sum_{t=1}^{T} e^{-it} \frac{\Delta_t}{2} (MarLoss_t + MarRecovery_t)$$

Coupon leg + Accrual on default

Tranche Pricing

Tranche	Contingent Leg	Coupon Leg (1)	Discounted Accrual on Default (2)	Fee leg≈DV01 (3)	Fair Spread
Equity	71.993%	2.18	0.09	2.27	31.725%
Mezz Jun	35.656%	3.68	0.05	3.72	9.573%
Mezz Sen	17.868%	4.14	0.02	4.17	4.288%
Senior Junior	9.052%	4.33	0.01	4.34	2.084%
Senior	2.478%	4.45	0.00	4.45	0.556%
Super Senior	0.030%	4.39	0.00	4.39	0.007%
Index	4.3081%	4.30	0.009	4.31	1.0000%

(4) The contingent leg is driven by the value of future recoveries.
The recovered amount is dependant on the magnitude of losses each period, and therefore the contingent leg is based on marginal losses only.

$$ContLeg = \sum_{t=1}^{T} e^{-it} MarLoss_t$$

(5)

$$S = \frac{Contigent\ Leg}{Fee\ Leg}$$

End: To ensure no arbitrage, the final index spread calculated using the tranche spread should be the same as the initial spread input

Gauss-Hermite integration

How to speed up the calculations

- In the first spreadsheet, the common factor M is modelled between -5 to +5 in steps of 0.1(see page 9) and assumed to follow a standard normal distribution
- Whilst this is a common approach for calculating a normal integral it is not the most efficient as it leads to 101 integration steps. Although not relevant at this stage, as we move to the later, more numerically intensive models, these large number of steps will have a significant impact on the calculation speed
- We therefore introduce the faster Gauss-Hermite technique in the second model. The model is set up identical to the first, with the only exception being the fewer (30) integration steps
- The Gauss-Hermite is a numerical method for approximating an integral using a limited number of points and is commonly used in practice for faster processing:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} e^{-x^2}\left[e^{x^2} f(x)dx\right] \approx \sum_{k=1}^{N} w(x_k)e^{x_k^2} f(x_k)$$

  - x_k is the random variable, which in our case is the common market factor M.
  - $f(x_k)$ is therefore a standard normal density function
  - $w(x_k)e^{x_k^2}$ is the weight given to each market factor (equivalent to the 0.1 integration step for the first model)
- The values used for the market factor and the correspondent weights depend exclusively on the number of points used for the integration, in our case 30

Gauss-Hermite integration

Spreadsheet screenshot

Numerical Integration

Gauss-Hermite: 30 point intergration (http://www.efunda.com/math/num_integration/findga

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} e^{-x^2}\left[e^{x^2} f(x)dx\right] \approx \sum_{k=1}^{N} w(x_k)e^{x_k^2} f(x_k)$$

> Common market factor
> Instead of having 101 integration steps from -5 to 5, we use 30 steps.
> The values chosen are derived from the number of steps and remain fixed

> Weight for each market factor. This is equivalent to the 0.1 step size used for model 1

		1	2	3	…	28	29	30	Sum
x_k	Common factor m	-6.9	-6.1	-5.5	…	5.5	6.	6.9	0.0
$w(x_k)e^{xk2}$	Weights	0.8342475	0.649098	0.5694027	…	0.569403	0.649098	0.83424747	14.6120994
$f(x_k).w(x_k)e^{xk2}$	Integral	1.965E-11	1.704E-09	5.108E-08	…	5.11E-08	1.7E-09	1.9651E-11	**0.99999999999986**

(Markers: 1 – common factor m; 2 – weights; 3 – integral sum; 4 – portfolio default distribution)

norminv(pd) = default barrier	E[pd(t)]	Check
-3.178	0.074%	0.000%
-2.580	0.494%	0.000%
-2.362	0.908%	0.000%

Portfolio default distribution, conditional on factor (m)

Time Period	1	2	3	…	28	29	30
1	45.150%	31.405%	21.565%	…	0.000%	0.000%	0.000%
2	70.790%	57.328%	45.310%	…	0.000%	0.000%	0.000%
3	78.537%	66.564%	54.988%	…	0.000%	0.000%	0.000%

> The conditional default distribution is calculated in exactly the same way as before, this time with the revised 30 market factors, and hence a smaller matrix
>
> $$PD(t|m) = \Phi\left(\frac{K(t) - \sqrt{\rho}\,m}{\sqrt{1-\rho}}\right)$$

> The integral is calculated identically to model 1
>
> P(M=m)=*f(m)**step size
>
> The sum of the integrals is extremely close to 1 indicating the degree of precision. Lower number of Gauss-Hermite factors will reduce the overall precision although it would result in faster computation

Disclosure appendix

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Issuer	Definition
Overweight	We expect the issuer to outperform sector peers over a 6-months horizon and would suggest holding more of the issuer's instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer's instruments and fundamentals compared to sector peers
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	All covered companies		Companies where a Dresdner Kleinwort company ha provided investment banking service	
Overweight	15	29%	9	60%
Marketweight	22	43%	7	32%
Underweight	14	27%	8	57%
Total	51		24	

Source: Dresdner Kleinwort Research

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